A Theory of Vibrations in Parachutes

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The basic modes of vibration of a tethered parachute are outlined and a simple theory for determining their fundamental frequencies is developed. All frequencies appear to be inversely proportional to linear dimensions and their various ratios can be defined in terms of the canopy mass/lines mass ratio and a function of the elastic properties of the material. Some discussion is given to the mechanism by which parachute lines pick up energy from high-speed air-flows. The importance of high tenacity fibres for lines of small heavy parachutes designed for high-speed use is demonstrated.

Nomenclature

C	= ratio $\{T/(dT/de)^{\frac{1}{2}}$ or $\{T_s(dT_s/de)^{\frac{1}{2}}$
C_1, C_2, C_3	= coefficients
e	= strain
f	= frequency of vibration (nonspecific)
f_s	= frequency of springing oscillation
f_t	= frequency of transverse whipping oscillation
f_l	= frequency of longitudinal strop oscillation
f_p	= frequency of pumping (or breathing) oscillations
f_c	= frequency of transverse vibration of rigging lines
g	= acceleration due to gravity
l	= length of elastic member in strop-line system
l_1	= length of cord over canopy
l_2	= length of rigging lines
m_c	= actual canopy mass
m_s	= strop-line mass
T	= tension
T_s	= specific tension
dT/de	= stretch modulus
dT_s/de	= specific stretch modulus
V	= true airspeed
V_i	= indicated airspeed
w	= weight per unit length
μ	= mass per unit length
σ	= air density ratio

Introduction

THE study of vibration in parachutes does not appear to have received much attention although several phenomena, such as breathing, line whip and ribbon flutter have been observed and commented upon. Some years ago, some violent instability of parachutes towed behind an aircraft[1] encouraged an examination of the subject in a general way to find whether vibration resonances and oscillatory phenomena were limiting factors in the design of parachute systems.

This paper will be primarily concerned with vibration phenomena as they affect the design of small high-speed parachutes because, for this area, a simple mechanical interpretation can have some practical relevance. The flat parachute will be considered as the basic model. However, the effects of such design modifications as shaping the gores or forming a conical canopy will be summarily considered. Sophisticated asymmetric designs are beyond the scope of the simple approach.

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Experimental Evidence on Vibrations in Parachutes

When small parachutes have been flown in wind tunnels[1,2] it has been noticed that they vibrate and breathe and that the predominant frequency is proportional to the tunnel speed. This is a result to be expected if the mode of vibration is a modified form of the transverse vibration of a heavy string and if the drag of the parachute is proportional to the square of the airspeed. The tension in each cord should be a constant fraction of the parachute drag and the frequency of vibration of a fixed length of cord is proportional to the square root of the tension. Thus it follows that the frequency is expected to be proportional to airspeed.

During the inflation of ribbon parachutes the peripheral ribbons flutter violently, which condition may persist, for certain high airspeeds, when the parachute has opened. The main mode of vibration of the ribbon is in torsion. Some incidental recordings have been made of the frequency of vibrating in the course of other work but there is no clear record of any published analysis in this field.

Principal Modes of Vibration

In specifying the principal modes of vibration of a parachute it will be assumed from the start that the parachute, attached to a relatively massive body, is made from rather extensible materials, such as nylon or polyester, and that the effective elasticity will be an important factor. Then there will be five modes of vibration that are of importance. a) The oscillation of a parachute as a mass on a spring (Figs. 1a, 1b) (elastic lines and strop). b) The longitudinal oscillation in the strop between terminal inertias. c) Transverse vibrations of the strop as a heavy cord (Fig. 1c). d) Transverse vibrations of the rigging lines as heavy cords (Fig. 1d). e) 'Breathing' oscillations of the canopy (Fig. 1e). In the aforementioned modes of oscillation, all but the first two have frequencies of vibration which are expected to be proportional to airspeed. The first two relate to elastic oscillations whose frequencies are substantially insensitive to airspeed. There will be slight variation with speed because the differential elasticity of the materials used is sensitive to the mean tension in them.[1]

In considering the major source of the elasticity for the first mode it is necessary to establish whether the line or the strop is the major contributor to the compliance of the system. In the towing tests which were originally analysed,[1] the parachute lines were relatively stiff and thus the whole parachute is treated as a mass on a spring. If the strop is made of a material like steel then most of the elasticity will occur in the parachute lines. If the lines are bridled into an extension strop then the total length of strop and line is relevant.

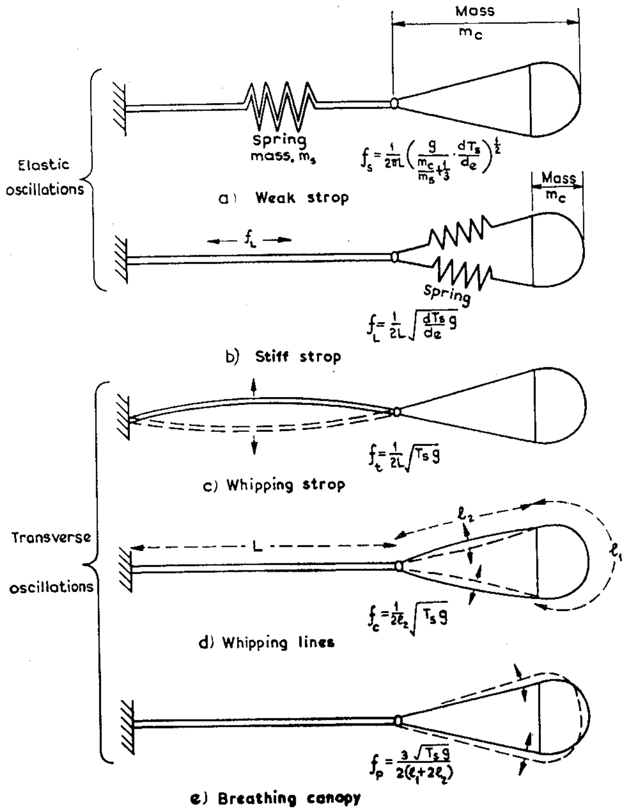

Fig. 1 Modes of vibration.

Tenacity as a Basic Parameter

In dealing with textiles it is customary to define the strength of a fibre in terms of its tenacity, that is the tension which will break the fibre divided by the weight of a given length, usually measured in Tex. Intermediate loadings are defined in terms of specific tension or specific stress. When fibres are made up into yarns, cords, webbing, ribbons and the like the effective tenacity of the manufactured product will be rather less than that of the fibre being reduced by a factor related to the geometry of the construction. In modern efficient constructions it is aimed to achieve at least 70% of the basic tenacity of the fibre. Thus if it is the aim to design a parachute with the same reserve factor on strength throughout there will be a proportionality between strength and weight.

The importance of this can be seen in the expression for the basic transverse and longitudinal frequencies of vibration on a tensioned cord of fixed length and held rigidly at each end. The classical study of the transverse vibration of heavy cords, as considered in most text books, relates to disturbances of infinitesimal magnitude for which it is shown that such disturbances propagate at a velocity determined by the square root of the ratio of the tension T to the mass per unit length, μ. If the length of the cord is l then the frequency f_t of vibration will be

$$f_t = (T/\mu)^{\frac{1}{2}}/2l \tag{1}$$

If w is the weight per unit length then $\mu g = w$ and Eq. (1) can be expressed in the form

$$f_t = (Tg/w)^{\frac{1}{2}}/2l \tag{2}$$

Now T/w is the quantity called specific tension which it is customary to determine for a fibre. Therefore, the same quantity, which will now be denoted by T_s, can be used for the cord and

$$f_t = (T_s g)^{\frac{1}{2}}/2l \tag{3}$$

Longitudinal oscillation will occur in the cord, similar to those in bars, for which the frequency f_l of vibration is given, in terms of the stretch modulus dT/de (the rate of change of tension T with respect to strain, e)

$$f_l = \{(dT/de)/\mu\}^{\frac{1}{2}}/2l = (dT_s g/de)^{\frac{1}{2}}/2l \tag{4}$$

in which equation the quantity dT_s/de is described as the specific stretch modulus.

Comparison of Transverse and Longitudinal Frequencies of Vibration of a Cord between Fixed Ends

The form of Eqs. (3) and (4) indicates that there is some advantage in using specific tension as a basic parameter. The ratio of the frequencies

$$f_t/f_l = \{T_s/(dT_s/de)\}^{\frac{1}{2}} \tag{5}$$

from which the length l, and the constant of gravity g, have been eliminated. This ratio C is a property of the material because it is fully defined by the stress-strain relationship of the material.

In general the specific tension T_s can be represented as a convergent polynomial expansion of e, thus

$$T_s = C_1 e + C_2 e^2 + C_3 e^3 + \dots \tag{6}$$

One can then write

$$f_t/f_l = (C_1 e + C_2 e^2 + C_3 e^3 + /C_1 + 2C_2 e + 3C_3 e^3 +)^{\frac{1}{2}} \tag{7}$$

which shows that the ratio of frequencies is always less than $e^{\frac{1}{2}}$ in a material with a concave stress-strain curve. In a perfectly elastic material C would be expressed by $e^{\frac{1}{2}}$.

Dynamic estimates of the stretch modulus can be calculated from direct measurement of the frequency with which a weight will perform vertical oscillations of a known length of cord.[1] The results of some measurements are given in Fig. 2.

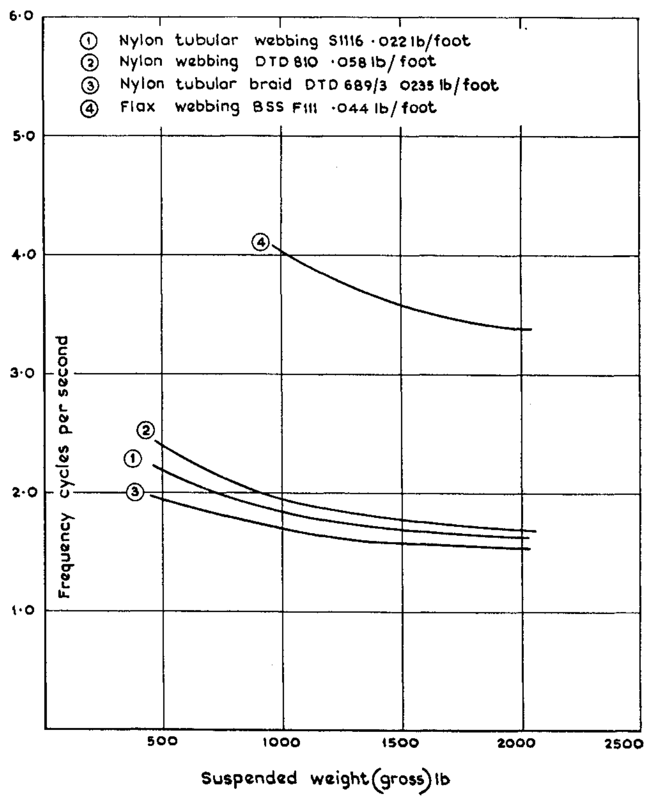

Fig. 2 Oscillation of weights on single thicknesses of webbing length 10 ft.

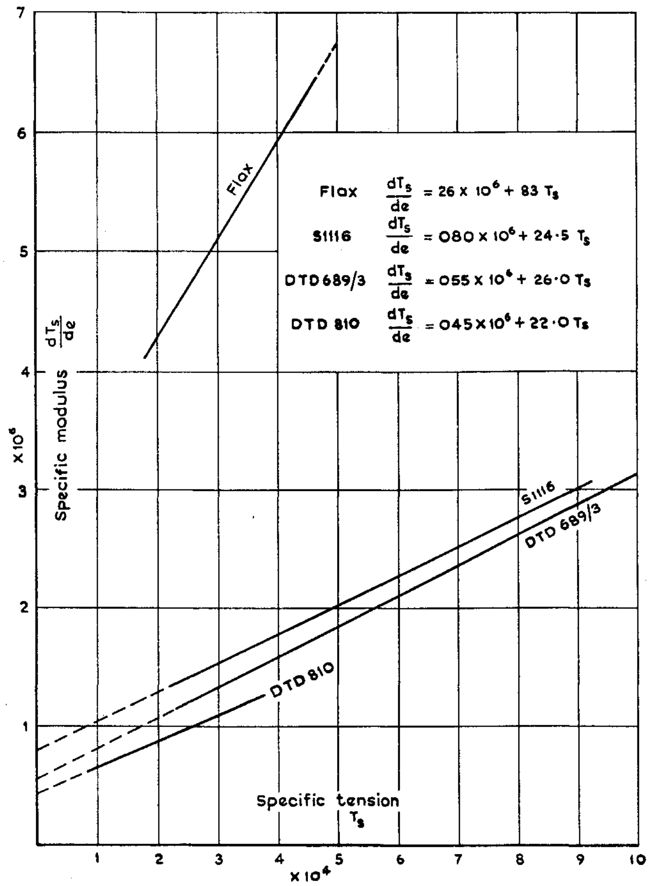

Fig. 3 Variation of specific modulus with specific tension for four materials.

The calculated values of specific stretch modulus, dT_s/de are plotted against the specific tension to Fig. 3. As the data show, there is an almost linear relationship between dT_s/de and T_s. For convenience the ratio C is plotted against the strain in Fig. 4 and is compared with the ideal curve for a perfectly elastic material.

With further reference to Fig. 4 it is seen that the longitudinal frequency is of the order of the fifth or fourth harmonic of the fundamental of the transverse frequency. It would be interesting to examine the interaction of the two types of motion through the second and higher order terms to see whether harmonic coupling is important when materials are so extensible. It could be serious if a harmonic interaction occurred near the design limit for loading a cord.

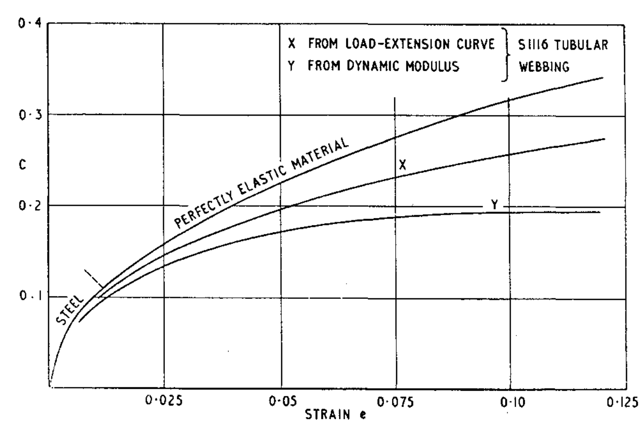

Fig. 4 Plot of $C = \{T_2/(dT_2/de)\}^{\frac{1}{2}}$ as a function of strain e.

Springing Oscillations on Store

These oscillations are basically responsible for the tension fluctuation. The motion of a heavy parachute canopy on heavy elastic lines has been considered where the fluctuations in the drag force can influence the motion.[3] This problem is regarded as one of a mass on a heavy spring in which the terminal mass is comparable with that of the spring. An example of the mass distribution of a parachute of 7ft 6in. flat diam, designed for a dynamic head of 12 psi (83 kN/m^2) is given in Fig. 5 from which it is seen that the canopy with canopy lines weighs 9.65 lb (43 N) compared with 7.85 lb (35 N) of free line. The weight of air contained within this canopy is only about 1.5 lb (6.5 N) so that the virtual air mass associated with it will be negligible.

Although there will be circumstances in which the assumption of constant effective mass for the canopy may not be valid, particularly when breathing is prominant, the simple model of a constant mass on a heavy spring will be taken as the first-order representation of this case. In defining the effective mass of the canopy it is important to make some allowance for the mass of the strop, one third of which should be added to the canopy mass in accordance with the approximate theory of the vibration of a mass on a heavy spring.[4] A more exact solution can be obtained[5] which shows that the approximate theory makes an allowance about 10% too great when the terminal and spring masses are equal.

The ratio of the stiffness of the strop or lines to the effective mass is equal to the square of the natural angular frequency of the system. In the present case the stiffness is represented by $(dT/de)/l$ and the effective mass is $m_c + m_s/3$ where m_c and m_s are, respectively, the canopy and spring masses. The undamped natural frequency f_s can be written

$$f_s = \{dT/de)/l(m_c + m_s/3)\}^{\frac{1}{2}}/2\pi \qquad (8)$$

In terms of specific stretch modulus dT_s/de one can write

$$f_s = \{(dT_s/de)\mu g/l(m_c + m_s/3)\}^{\frac{1}{2}} \qquad (9)$$

but $m_s = \mu l$, so that

$$f_s = \{(dT_s/de)/(m_c/m_s + \tfrac{1}{3})\}^{\frac{1}{2}}/2\pi l \qquad (10)$$

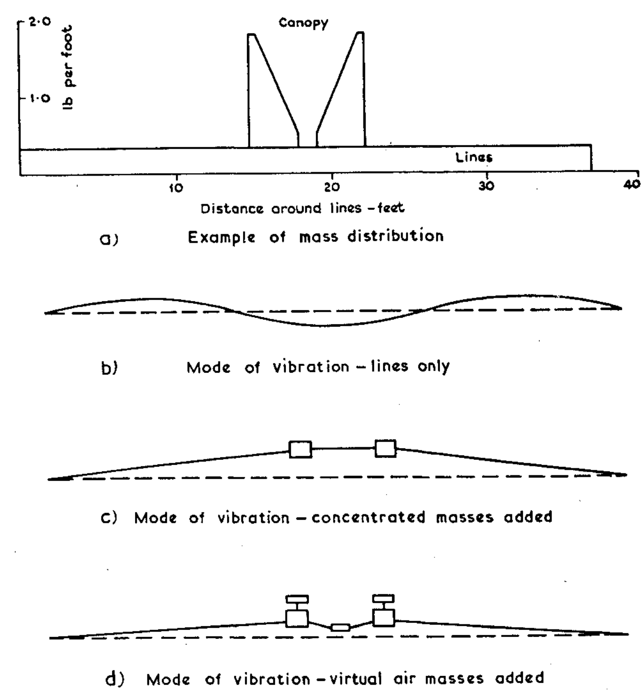

Fig. 5 Modes of Vibration due to distributed mass.

from which it is seen that l now comes outside the square root and is in a form comparable with Eqs. (3) and (4).

The Relation Between Transverse and Springing Frequencies of a Simple System

The ratio of the springing frequency f_s to the transverse frequency f_t in the simple system examined is of particular interest. It is given by

$$f_t/f_s = \pi\{(m_c/m_s + \tfrac{1}{3})T_s/(dT_s/de)\}^{\frac{1}{2}} \qquad (11)$$

It is seen that when the canopy mass is sufficiently heavy f_t can be equal or greater than f_s, but f_t varies with speed and f_s does not, so that it is conceivable that a system is designed for which at some speed these two frequencies are equal.

Breathing or Pumping Oscillations

In small heavy parachutes designed for high-dynamic pressures it can be shown, for flat parachutes at least, that the mode of oscillation is a modified form of a transverse cord oscillation. In this case the heavy cord itself is the predominant factor in determining the mode of vibration and a simple calculation can be made for this case. However, the added distributed mass of the canopy will tend to lower the frequency of vibration as will also associated air masses which will tend to concentrate their effects at the hem and the crown. On the other hand, constraining the parachute by shaping the gores or by removing a few gores to form a flat cone, can introduce some constraint to a pumping displacement tending to raise frequencies.

A parachute cord is almost in uniform tension over the canopy so it is possible to obtain an estimate of the frequency from the simple theory of transverse vibration in heavy cords. The eye of the rigging is a node and two other nodes occur near the hem of the canopy. Thus, for a sinusoidal oscillation a standing wave of one and one half wave length is formed in the cord. If l_1 and l_2, respectively, represent the length of cord over the canopy and the free line length, then the pumping frequency f_p is given by

$$f_p = 3T_s^{\frac{1}{2}}/2(l_1 + 2l_2) \qquad (12)$$

In comparison, the frequency f_c for the transverse vibration of the free rigging line is given by

$$f_c = T_s^{\frac{1}{2}}/2l_2 \qquad (13)$$

These frequencies will be the same when $l_1 = l_2$ but since l_2 is often made twice l_1 in these small parachutes this would make $f_p > f_c$ and place the nodes outside the canopy. Thus the breathing frequency of a small heavy parachute is of the same form, and of much the same value, as the transverse frequency of the free lines. The component modes of vibration are illustrated in Figs. 5b–d.

The Trends of Frequency Variation

There are two particularly interesting observations to be made on this simple theory of vibrations. Firstly, the frequencies of all the modes considered are inversely proportional to the linear scale of the parachute and, secondly, that the ratios of the frequencies depend upon explicit factors, one describing the elastic properties of the material and the other the mass distribution between canopy and lines. The ratio of the transverse frequency to the springing frequency of a simple system representing the parachute and lines shows that this ratio could be unity. If this condition led to cross-coupling and a large amplitude breathing oscillation the validity of considering the canopy as a concentrated mass is in doubt. The trend would be to reduce the apparent mass of the canopy and move off the resonant condition.

Table 1 Vibration data for two parachutes

	Lightning Brake Parachute Mk. 3	Vehicle recovery parachute
Flat diameter	16 ft (4.88 m)	7 ft 6 in. (2.28 m)
Design speed	370 fps (112.5 m/s)	1200 fps (365 m/s)
Net parachute weight	15.65 lb (66.5 N)	17.5 lb (78 N)
Canopy/line mass ratio	1.77	1.23
Design specific tension	6.4×10^4 ft (19.5 km)	5.8×10^4 ft (17.6 km)
Ratio C	0.167	0.161
Frequency ratio at design speed	0.765	0.63

The frequency of the transverse cord vibrations and of breathing will be below that of the springing oscillation in most designs. These frequencies will depend upon the airspeed increasing in many cases proportionally with airspeed to be nearest to the springing frequency at the design speed. It is important that no interaction of the modes of vibration occurs at the design speed because this might raise the peak stresses for the case of greatest mean stress.

Two actual designs have been examined to see how they reckon against the criteria of the simple theory. The canopy/lines mass ratio has been determined for two British parachutes, both conventional flat ribbon parachutes.

It is seen from Table 1 that resonance conditions are not far away at the design speeds. Moreover, the design specific tension corresponds to only 4–5% mean strain in a system in a material that has an ultimate of about 20%. The insertion of a more compliant strop than the lines into the system can make matters worse and this was the cause of severe resonance difficulties in the early flight trials.[1] This resonance problem raises the question of the relative merits of the flat, conical and 'hemisflo' parachutes in this respect. Apart from the known aerodynamic disadvantages of the flat parachute at transonic and supersonic speeds[6] this design is prone to pumping. This is understandable because there is no change of internal volume for small displacements. Over-inflation also occurs on opening which applies extra loads to the parachute. With conical and 'hemisflo' parachutes the increased peripheral tensions in such designs should give more restraint to the transverse motion of the cords and, in turn, raise the whipping and pumping frequencies to be closer to the springing frequency. This is a factor for experimental study.

The conclusion is that there exists a problem of vibration analysis in parachutes which requires examining more rigorously. It is necessary to establish clearly whether identical breathing and springing frequencies can be designed into a parachute and whether they interact. There are circumstances where it is necessary to increase the efficiency of a design by working to higher mean stress levels and relieving shock loads by the use of ply-tear textiles[7,8] to take out the surplus energy. The aims would be defeated if a resonant condition which reinforces the peak loads was introduced.

The Forcing of Vibrations

A description of modes of vibration and their natural frequencies is incomplete without some indication of how they are forced. The springing oscillation is forced by the mass of the canopy being stopped on the elastic lines. These have been observed in man-carrying parachutes.[3] At normal density and pressure of the air these oscillations are usually well-damped. Breathing oscillations can be forced mechanically by over-inflation on opening. These have been illustrated from the early experiments on the relaxation catapult.[9]

Although the maintained forcing of breathing oscillations, which probably arise from aerodynamic sources, is not well understood, one particular theory of forcing associated with high-speed parachutes is discussed. When the specific tension in a line is insufficiently high in relation to the air drag on it a state can arise in which energy is picked up from a steady air flow. Phillips,[10] some years ago, showed that, in a cable inclined to air flow, this would occur when the tangential airspeed was faster than speed of travel of transverse waves. However, he did not consider a cable of finite length but it is apparent from his work that such a cable would settle to a finite amplitude of vibration. This phenomenon could well arise with the heavy rigging lines of high-speed parachutes.

The significance of the criterion of Phillips in defining the minimum specific tension in a cable to ensure stability was pointed out in connection with towing small parachutes at high speed behind aircraft.[11] This can be restated as

$$V^2 < T_s g \tag{14}$$

In terms of indicated airspeed V_i and the air density ratio σ the inequality is

$$V_i^2 < \sigma T_s g \tag{15}$$

This shows that as the speed is increased the specific tension in the cords must be increased and finally a stage is reached when the tenacity of real materials is not good enough and it is predicted that some measure of forced vibration must be accepted. A plot of T_s against V_i is given in Fig. 6 in both metric and imperial units.

Conclusions

The analysis of this paper has pointed to certain trends in the vibration characteristics of parachutes by making use of simple mechanical models. All frequencies appear to be inversely proportional to linear dimensions and their various ratios can be defined in terms of the canopy mass/lines mass ratio and a function of the elastic properties of the material. It would appear that parachutes can be designed in which two entirely different modes of vibration, that is the breathing and springing modes, could have frequencies of the same value. However, the treatment of this paper would not be valid for a resonant condition and there is a need for more analysis using a better model of the parachute to discover whether resonant conditions exist.

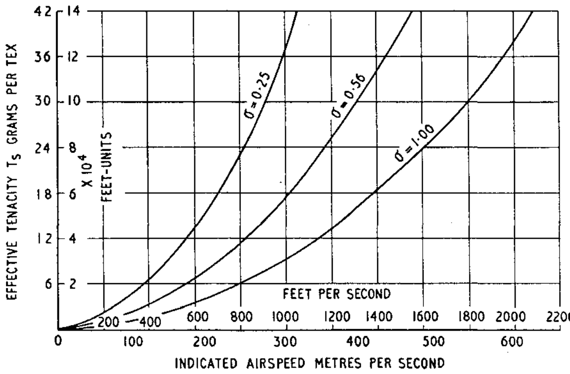

Fig. 6 Relationship between effective tenacity of cords and indicated airspeed for critical condition $V_i^2 = \sigma T_2 g$.

Guidelines for Future Experimental Research

Some useful work can be done by gleaning relevant data from application trials where a parachute has been observed to vibrate and the frequency can be determined by cine or other means. However, it is necessary to collate also the mass distribution in the parachute and the elasticity of the system. If special trials are done it would be valuable to compare parachutes of the same canopy design but with different line lengths and also to compare parachutes of identical design but of different size. The dynamic elastic modulus, of parachute lines can be conveniently determined by bouncing a weight on a length of line. This is a simple laboratory experiment. The natural breathing frequency of a flat parachute, at least, can be estimated from the breathing oscillations induced by overinflation when a parachute is first opened.

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